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THE PROCESS OF SEPARATION IN THE
TURBULENT FRICTION LAYER

By E. Gruschwitz

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THE PROCESS OF SEPARATION IN THE TURBULENT FRICTION LAYER*

By E. Gruschwitz

The separation of the flow which occurs at large angles of attack on the suction side of an airplane wing is attributable to phenomena in the flowing fluid layer adjacent to the surface; the fluid particles slowed up by the friction on the surface can no longer advance against an unduly great pressure rise. It is of vital importance that there exist two types of flow - laminar and turbulent - in the fluid layer flowing in the immediate vicinity of a body. According to Prandtl, by whom the whole theory was developed, we speak in the first case of a laminar boundary layer, in the second, of a turbulent friction layer.

Beginning at the stagnation point, there is always a laminar boundary layer present, which at some distance from the stagnation point changes into a turbulent friction layer on sufficiently large bodies and by sufficiently high speeds. In view of the overcoming of a rise in pressure, the turbulent flow attitude is markedly more favorable than the laminar attitude, since the turbulent mixing effects a much more intensified impulse transfer on the decelerated fluid particles than the mere viscosity.

I. METHOD FOR APPROXIMATE CALCULATION OF FRICTION LAYER

In another report** the writer attacked the problem of making the turbulent friction layer amenable to calculation for the case of steady plane flow. Being practically ignorant of the nature of the turbulent isolated processes it, of course, pertained only to an approximation method derived from experiments. The method is briefly, as follows:

*"Über den Ablösungsvorgang in der turbulenten Reibungsschicht." Z.F.M., June 14, 1932, pp. 308-312.

**Ingenieur-Archiv II (1931), p. 321.

The premises of all boundary-layer calculations are that the speed at relatively short distance from the surface becomes the speed of a potential flow and that within the friction layer the static pressure perpendicular to the surface does not change. Admittedly, only the mean values with respect to time in the turbulent flow are of interest so long as it is not desired to follow the turbulent isolated motions. The velocity distribution in the friction layer past a flat plate exposed longitudinally to a flow, that is, in the absence of any pressure gradient, can be interpolated with close approximation by power formula. (Reference 1.) For velocity distributions which reveal the friction layers in the presence of greater pressure gradients along the body, such interpolation is impossible, as Figure 1 shows. Besides, it appears hopeless of finding a practical mathematical term for a set of curves whose individual curves would reproduce the velocity distributions. One therefore must forego such an interpolation.*

One first legitimacy, which must suffice the friction layer, is supplied by the momentum theory, as first applied by v. Karman to boundary layers. The derived equation reads:

$$\frac{d\vartheta}{dx} + \frac{H+2}{2} \frac{\vartheta}{q} \frac{dq}{dx} = \frac{\tau_0}{\rho W^2} \quad (1)$$

where

$$\vartheta = \int_0^\infty \frac{(W-w)w}{W^2} dy \quad \text{and} \quad H = \frac{1}{\vartheta} \int_0^\infty \left(1 - \frac{w}{W}\right) dy$$

(x = length of arc at surface of body in direction of flow, y = coordinate at right angle to it, W = magnitude of potential flow, $q = \frac{\rho W^2}{2}$, w = speed in friction layer, τ_0 = shear stress at wall and ρ = density).

*In a recently published article (Horst Müller, "Frictional Resistance of Bodies in a Flow," *Werft, Reederei, Hafen*, 1932, No. 4, p. 54), the velocity profiles in friction layers by pressure rise and drop are approximated by the formula $\frac{w}{W} = \left(\frac{y}{\delta}\right)^{1/7}$ (δ = distance in which the friction layer changes to potential flow, and conclusions drawn (after the pattern of v. Karman, loc. cit.) for the friction layer on a flat plate. It yields a practical estimate of the frictional resistance in comparatively simple manner. Since a solid profile form is assumed, it yields, of course, no way to the separation problem.

The quantity δ , having the dimension of a length, can be looked upon as a criterion for the thickness of the friction layer; $\delta \rho W^2$ is the loss of momentum due to friction in the friction layer. In the absence of this layer, that is, in the presence of the potential velocity W directly on the surface, the flow of momentum between the surface and a point at distance δ in unit of time would be as great as the loss through friction in the friction layer.

Equation (1) is essentially a differential equation for δ , with, however, H and $\tau_0/\rho W^2$ as further unknown quantities. Compared to the other terms of the equation, on the other hand, quantity $\tau_0/\rho W^2$ is quite small when the pressure gradient

$$\frac{dp}{dx} \left(= - \frac{dq}{dx} \right)$$

assumes an appreciable value. Consequently, it suffices to substitute a constant approximation value. It was found that $\tau_0/\rho W^2 = 0.002$ was quite acceptable. Quantity H shall be referred to again later.

It being important to investigate the processes which lead to separation of the friction layer, it does not suffice to follow up its thickness, but rather to observe the form of the velocity profiles. We concentrate on the velocity at a point of the friction layer lying near the wall. When this velocity drops below a certain value as compared to the velocity outside of the friction layer, it may be conceded that separation is imminent. To define this point we are without all means except that selected for the thickness of the friction layer and, in particular, it has proved advisable to choose the point at distance δ from the wall. δ is quite small compared to the width taken up by the friction layer; for example, by a velocity distribution of $\frac{W}{W} = \left(\frac{y}{\delta}\right)^{1/7}$ $\delta = \frac{7}{72} \delta$ (the approximation formula for the flat plate).

The fluid particle at this point, whose velocity may be designated at w_1 , can be expressed by an approximate motion equation. The turbulent motions transfer impulses from the outside to the particles and give off impulses on the fluid layer farther inside. Of these impulses it

is assumed that they are determined by w_1 , W , ϑ and density ρ (possibly the viscosity might be of influence, but in turbulent motions it is mostly negligible compared to the turbulent momentum exchange). Furthermore, it should be borne in mind that the curve $y = \vartheta$, on which the fluid particle is visualized as moving, has practically the aspect of a streamline. Then the equation of motion reads:

$$\rho w_1 \frac{dw_1}{dx} + \frac{dp}{dx} = f(w_1, W, \vartheta, \rho) \quad (2)$$

(p = static pressure) or, putting $\frac{\rho w_1^2}{2} + p = g_1$

$$\frac{dg_1}{dx} = f(w_1, W, \vartheta, \rho) \quad (3)$$

Since every physical relation must lend itself to being expressed as equation between dimensionless quantities, and inasmuch as ~~the~~ the five quantities appearing in (3) only two permit the formation of independent dimensionless quantities, equation (3) can also be written in the form

$$\frac{\vartheta}{q} \frac{dg_1}{dx} = F\left(\frac{w_1}{W}\right) \quad (4)$$

Measurements of the velocity distribution in friction layers by pressure rise and pressure drop have actually revealed in close approximation a relationship of form (4), which may be interpolated by the equation

$$\frac{\vartheta}{q} \frac{dg_1}{dx} = 0.00894 \eta - 0.00461 \quad (5)$$

whereby

$$\eta = 1 - \left(\frac{w_1}{W}\right)^2 \quad (6)$$

(Fig. 2.) It is seen from (5) that the turbulent motions extract more momentum from the fluid particle with velocity w_1 than it obtains from without when

$$\eta < \frac{0.00461}{0.00894} = 0.516,$$

whereas the momentum from without preponderates when $\eta > 0.516$, that is, so much more as η is greater, or in other words, as the particles remained behind relative to the potential flow. Equation (6) can also be written:

$$\eta = 1 - \frac{\xi_1 - p}{q}$$

or

$$q \eta = [q + p] - \xi_1$$

Since $q + p$ is a constant quantity, it follows:

$$\frac{d\xi_1}{dx} = - \frac{d(q\eta)}{dx},$$

so that (5) can be written as

$$\delta \frac{d(q\eta)}{dx} = - 0.00894 q \eta + 0.00461 q \quad (7)$$

Visualize w/W plotted against y/δ and assume that the thus produced sheaf of curves is approximately of one parameter, so that quantity w_1/W or η in its stead, can be considered as sheaf parameter. Then the quantity

$$H = \frac{1}{\delta} \int_0^{\infty} \left(1 - \frac{w}{W}\right) dy = \int_0^{\infty} \left(1 - \frac{w}{W}\right) d\left(\frac{y}{\delta}\right)$$

occurring in (1) must be a function of η , as is actually the case according to the experiments. (See fig. 3.) Between H , δ and the length

$$\delta^a = \int_0^{\infty} \left(1 - \frac{w}{W}\right) dy$$

sometimes used as criterion for the thickness of the friction layer, there exists moreover, the relationship $H = \delta^a/\delta$, as seen from the definition of H .

The result is a system of differential equations in (1) and (7) from which by known velocity past a body and by known initial values the friction layer can be computed, because η defines the form and δ and the prescribed potential velocity W , the expanse of the velocity profile. The presumption hereby is that the thickness of the friction layer is small compared to the curvature radius of the surface, because (7) has been derived from measurements on a flat wall, whereby the rise and drop in pressure had been produced by corresponding formation of an opposite wall. On markedly curved walls a certain curvature effect is to be expected according to the experiments which are still being continued. As concerns the initial

values, they stipulate an idealization, if the turbulent friction layer is to be computed from the beginning. It is assumed that the change from laminar boundary layer into turbulent friction layer occurs with a sudden jump. In reality, of course, this change-over extends over a great distance, but it has been found that this idealization is very expedient when placing the point of transition where the boundary layer begins to become turbulent, and there to give the turbulent friction layer the value $\eta = 0.1$. As initial value for δ , one must select the value which the laminar boundary layer exhibits at its end, to ensure steady impulse content.

A close approximation for δ can be obtained by substituting constants (say,

$$H = 1.5, \quad \frac{\tau_0}{\rho W^2} = 0.002),$$

for H and $\frac{\tau_0}{\rho W^2}$ in equation (1) which, thus becoming a linear differential equation of the first order for this approximation, is inserted in (7) to yield a linear differential equation of the first order for q η . After computing this, the course of H can be ascertained and written in (1), which yields a linear differential equation of the first order for a second approximation of δ for computing a second approximation of η , etc. The resolution of the linear differential equations is best done by E. Czuber's graphical method. (Reference 2.) In the examples computed heretofore, it was never necessary to go beyond the second approximation. In many cases even the first showed satisfactory agreement with the experiments. Separation is expected when $\eta > 0.80$.

The objectionable feature of this method is that it stipulates the knowledge of the laminar boundary layer and of the point of transition into turbulence. For example, having computed the flow around an airplane wing according to the potential theory, it is necessary to compute the laminar boundary layer, which can be effected approximately by the Karman-Pohlhausen method. (Reference 3.) The point of transition is primarily contingent upon the value of $R = W\delta/\nu$, that is, on a Reynolds Number formed with the quantities decisive for the expansion of the velocity profile. However, no exact figure of R , at which the boundary layer becomes turbulent, can be given. Obviously, it also depends on the form of the laminar velocity profiles and undoubtedly also on the prop-

erty of the air stream to which the airplane wing is exposed. According to measurements hitherto, the laminar boundary layer obtains up to $R = 650$. If return flow occurs before reaching this figure, the transition to turbulence occurs at the point of inception of this return flow, provided R has there exceeded a certain figure which, although not definitely determined, should be below 250, since that is the lowest figure for R observed in all turbulent friction layers up to now. If the boundary layer evinces no sufficiently high R at the point of incipient return flow, there is no formation of turbulent friction layer and separation occurs.

II. INFLUENCE OF REYNOLDS NUMBER ON FRICTION LAYER

It is remarkable that the Reynolds Number $R = W_0 \delta / \nu$ does not occur in equation (7). For its derivation we employed measurements in friction layers in which $R = 250$ to 5,450 occurred. Whether or not there is a relationship of R in experiments on friction layers with higher Reynolds Numbers, is an open question, though improbable because of the already mentioned subordinate role which viscosity plays in comparison with the turbulent motions. In (1) the influence of the Reynolds Number is confined to the value of $\tau_0 / \rho W^2$. But this influence is quite small, and $\tau_0 / \rho W^2$ is very small compared with the other two terms when dp/dx is great, as is the case in flows in which separation is in question, so that as a result, the dependence of this equation on R can be ignored in such flows, and the turbulent friction layer can be considered as being independent of R .

Nevertheless, the total process is not independent of the Reynolds Number $R_t = \frac{W_0 t}{\nu}$ which, as customary, is formed with the flow velocity W_0 and a characteristic length t of the exposed body (as with the chord, say, on the airplane wing). The laminar boundary layer, always present at the beginning is, of course, dependent on the viscosity and consequently, on any formulated Reynolds Number; specifically, δ/t is proportional to $\frac{1}{\sqrt{R_t}}$ (reference 4), according to boundary layer theory. The relative thickness of the boundary layer at a certain point decreases as R_t increases, whereas the value of $R = \frac{W_0 \delta}{\nu}$

increases at this point when R_t increases, because δ/t proportional to $\frac{1}{\sqrt{R_t}}$ means that $R = \frac{\delta W}{\delta}$ is proportional to $\frac{t W}{v \sqrt{R_t}}$ that is, proportional to $\sqrt{R_t}$. Accordingly, as R_t increases, the point of transition to turbulence moves forward, while on the contrary, the friction layer begins with a smaller δ value. Equation (5) reveals that the impulse given a particle with velocity w_1 , positive when $\eta > 0.516$, and thus aiding to overcome the pressure rise, is inversely proportional to δ , so that a smaller δ value is more favorable in the sense of avoidance of separation ($\eta > 0.516$, ordinarily by pressure rise). The shifting of the transition point acts in the reverse sense. The farther forward the beginning of the turbulent friction layer, the greater the pressure rise it has to overcome and the more imminent the danger of separation.

In this way one can visualize the manner in which the dependence of the maximum lift of an airplane wing on the Reynolds Number R_t comes into being. It may be assumed that the maximum lift occurs when the separation begins at the trailing edge or is directly imminent there. In wings with well-rounded-off leading edges and fairly thick profile, the turbulent transition occurs considerably behind the pressure minimum at lower Reynolds Numbers. For higher Reynolds Numbers the transition point shifts forward and the flow separates therefore at lower angles of attack, i.e., the coefficient of maximum lift becomes smaller. Lastly, by further increase in Reynolds Number the point of transition finally falls in the vicinity of the pressure minimum, where it probably remains or at least shifts very little, because in the zone ahead of it, the boundary layer will be so thin by pronounced pressure drop as to scarcely induce turbulence even by very high Reynolds Numbers. Then the influence of the diminution on δ would have to make itself felt, and an increase in maximum lift is surmised. In wings with sharp leading edge, that is, such with thin profile, especially, the leading edge evidently produces the transition to turbulence, and since it then always remains at the same place, only the diminution of δ is of influence by an increase in Reynolds Number, so that a slight increase in maximum lift coefficient is to be expected. As a matter of fact, comparative measurements at different Reynolds Numbers such as those

made at Göttingen* and in the variable density wind tunnel of the National Advisory Committee for Aeronautics (reference 5), reveal that the maximum lift coefficient ordinarily decreases by increasing Reynolds Number for thick profiles with well-rounded-off leading edges, and a slight increase for thin profiles. But for unobjectionable proof of the surmise, according to which the coefficient of maximum lift should increase at high Reynolds Numbers even for thick profiles, the measurements made heretofore are insufficient.

A very similar process occurs on cylinders and spheres also. Admittedly, equations (1) and (7) are invalid for the sphere, in so far as it pertains to a symmetrical rotational flow. But qualitatively the expected behavior of the friction layer is similar. Resistance measurements on cylinders and spheres reveal that beginning from the critical value their resistance coefficient increases. (Reference 6.) By "critical value" is meant the value of the Reynolds Number $W_0 d / \nu$ (W_0 = flow velocity, d = diameter) at which a turbulent friction layer occurs. The reason for this is, according to the above, the forward displacement of the transition point and with it its point of separation by increasing Reynolds Number, with a resultant enlargement of eddy zone and consequently, resistance.

Translation by J. Vanier,
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for Aeronautics.

*See Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, Report I, p. 54.

REFERENCES

1. Karman, Th. v.: Zeitschrift für angewandte Mathematik und Mechanik, No. 1, 1921, p. 233.

van der Hegge Zijnen, B. G.: Measurements of the Velocity Distribution in the Boundary Layer along a Plane Surface. (Thesis) Delft, 1924.

Hansen, M.: Zeitschrift für angewandte Mathematik und Mechanik, No. 8, 1928, p. 185.
2. Czuber, E.: Zeitschrift für Mathematik und Physik, No. 44, 1899, p. 41.

Hort, W.: Die Differentialgleichungen des Ingenieurs. Berlin, 1925, p. 237.
3. Pohlhausen, K.: Zeitschrift für angewandte Mathematik und Mechanik, No. 1, 1921, p. 252.
4. Tollmien, W.: Artikel über Grenzschichttheorie im Handbuch der Experimentalphysik, IV, No. 1, Leipzig, 1931.
5. Seiferth, R., and Betz, A.: Untersuchung von Flugzeugmodellen im Windkanal. Handbuch der Experimentalphysik, IV, No. 2, Leipzig, 1932.
6. Muttray, H.: Die experimentellen Tatsachen des Widerstandes ohne Auftrieb. Handbuch der Experimentalphysik, IV, No. 2, Leipzig, 1932.

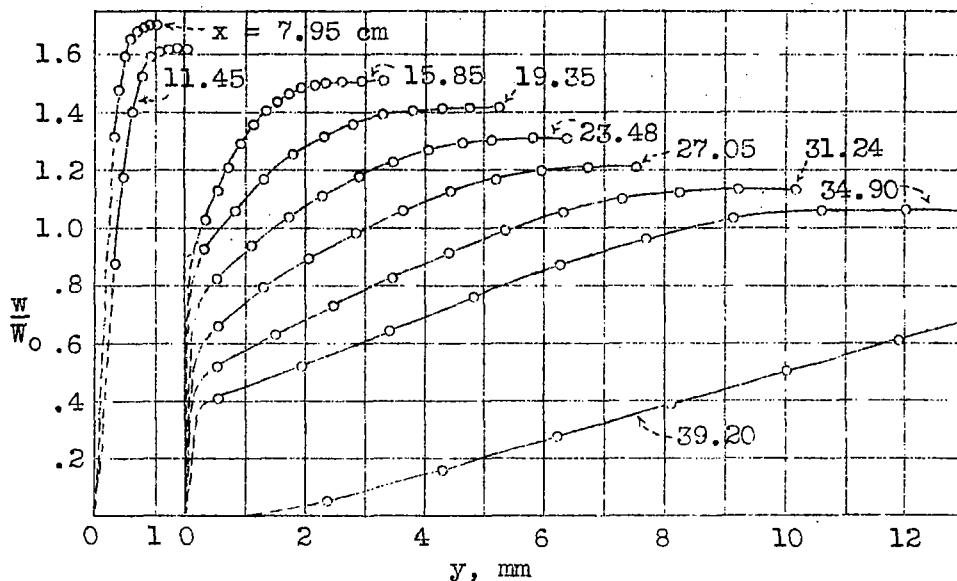


Fig. 1 Velocity profiles on Göttingen airfoil No. 387 (chord = 40 cm), flow velocity $W_0 = 30.8$ m/s, angle of attack $\alpha = 12^\circ$, x = length of arc along suction side from foremost point (total length to trailing edge = 42.7 cm), y = distance from surface, w = velocity in boundary layer or friction layer. The first two velocity profiles are laminar, the rest, turbulent.

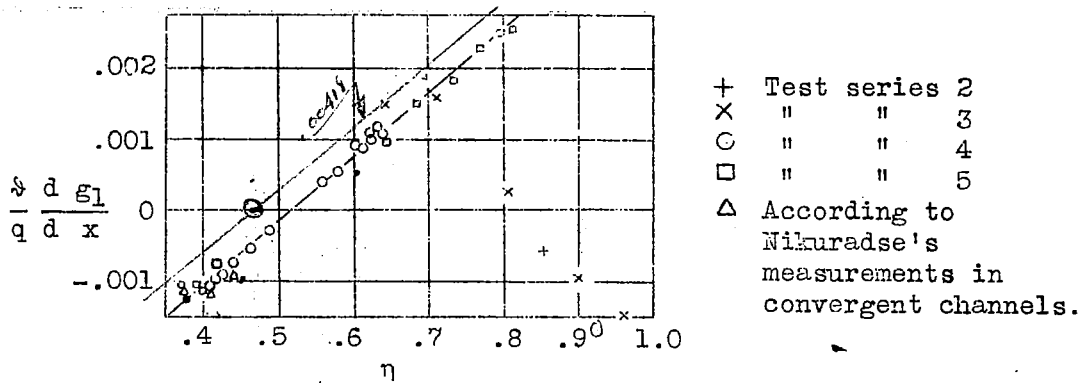


Fig. 2 $\frac{d}{q} \frac{d \xi_1}{d x}$ versus $\eta = 1 - \left(\frac{w_1}{W} \right)^2$

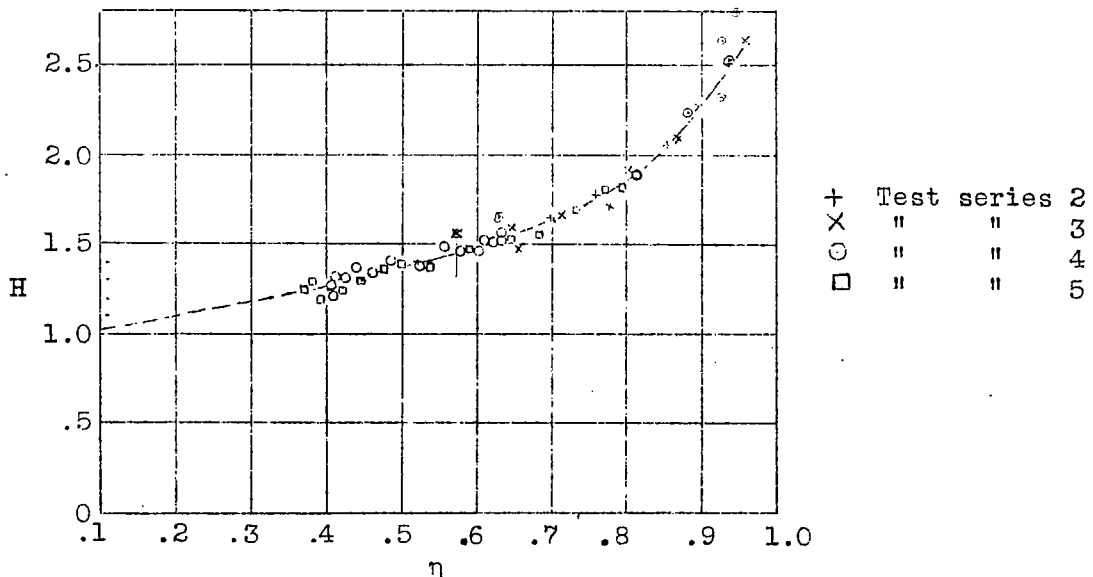


Fig. 3 $H = \int_0^{\infty} \left(1 - \frac{w}{W} \right) d \left(\frac{y}{\delta} \right)$ versus $\eta = 1 - \left(\frac{w_1}{W} \right)^2$

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